MULTIMEDIA UNIVERSITY

STUDENT IDENTIFICATION NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2016/2017 SESSION

BMS2024 - ADVANCED MANAGERIAL STATISTICS

(All Sections / Groups)

13 OCTOBER 2016
9am – 11am
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of 11 pages excluding the cover page.
2. This question paper consists of **FOUR** structured questions. Attempt **ALL** questions.
3. Students are allowed to use non-programmable scientific calculators with no restrictions.
4. Statistical tables are attached at the end of the question paper.
5. Please use **pen** to write the answers.
6. Please write all your answers in the answer booklet provided.

QUESTION 1

a) Suppose the judge decides to acquit all defendants, regardless of the evidence. What is the probability of a Type I error? Why might this be a bad policy? [4 marks]

b) Suppose a firm decides to test its employees for illegal drug usage. State the null and alternative hypotheses. Define type I and II error in this case? Which type of error is more to be feared by the employees and the employer. Explain your reasons. [4 marks]

c) The Federal Drug Administration (FDA) often test claims, that companies make about their products. Suppose coffee is being sold in cans that are labeled as containing 3 pounds and that the FDA also wishes to determine if the mean amount of coffee in all such cans is at least 3 pounds. To do this, the FDA tests a sample of 35 coffee cans assuming the population standard deviation is known to be 0.0147 pounds. Calculate the probability of a Type II error if the true mean is in fact 2.995 pounds. Compute and explain the power of the test. [17 marks]

[Total : 25 Marks]

QUESTION 2

a) Explain the differences between parametric and nonparametric tests. Discuss in terms of their assumptions and the circumstances under which each should be applied. [5 marks]

b) The Financial Budget Committee in a company, is interested in possible differences in days worked by salaried employees in three departments, namely, Sales, Accounts, and Administration. The table below shows annual days worked by 23 randomly chosen employees from these departments. Because the sampling methodology reflects the department sizes, the sample sizes are unequal. The distribution of the data is skewed to the right. Be sure to show all steps and workings. [20 marks]

Continued...

Sales	Accounts	Administration
245	205	233
258	217	233
260	220	240
265	228	242
278	239	244
	240	249
	240	256
	255	258
	266	
	270	

[Total : 25 Marks]

QUESTION 3

The president of a company that manufactures drywall wants to analyze the variables that affect demand for his product. Drywall is used to construct walls in houses and offices. Consequently, the president decides to develop a regression model in which the dependent variable is monthly sales of drywall (in hundreds of 4 x 8 sheets) and the independent variables are number of building permits issued in the country, five year mortgage rates (in percentage points), vacancy rate in apartments (in percentage points), and vacancy rate in office buildings (in percentage points).

To estimate a multiple regression model, he took monthly observations from the past 2 years and generated the following table, given in the next page.

Continued...

A	B	C	D	E	F
SUMMARY OUTPUT					
Regression Statistics					
Multiple R R Square Adjusted R Square Standard Error Observations	0.9453 0.8935 0.8711 40.13 24				
ANOVA					
	df	SS	MS	F	Sig. F
Regression	4	256,793	64,198	39.86	0.0000
Residual	19	30,602	1,611		
Total	23	287,395			
	Coefficients	Standard Error	t Stat	P - value	
Intercept	-111.83	134.34	-0.83	0.4155	
Permits	4.76	0.395	12.06	0.0000	
Mortgage	16.99	15.16	1.12	0.2764	
A.Vacancy	-10.53	6.39	-1.65	0.1161	
O. Vacancy	1.31	2.79	0.47	0.6446	

a) Compute the multiple regression equation model? [5 marks]

b) What is the coefficient of determination, and what does it tell you about the regression model? [5 marks]

c) Test the overall validity of the model? [5 marks]

d) Interpret each of the coefficients? [5 marks]

e) Predict next month's drywall sales if the number of building permits is 50, the 5 year mortgage rate is 9.0%, and the vacancy rates are 3.6% in apartments and 14.3% in office buildings? [5 marks]

[Total: 25 marks]

Continued...

QUESTION 4

Integrated circuits are manufactured on silicon wafers through a process that involves a series of steps. An experiment was carried out to study the effect of the cleansing and etching steps on the yield. The results are as follows together with the Two-way ANOVA output from EXCEL.

	ETCHING STEP	
CLEANSING STEP	**New**	**Standard**
New 1	38	34
	34	19
	38	28
New 2	29	20
	35	35
	34	37
Standard	31	29
	23	32
	38	30

Two-way ANOVA output from EXCEL

Source of Variation	df	SS	MS	F	P-value	F Critical
Sample	2	6.333333	3.166667	0.087289	0.916988	3.88529
Columns	1	72	72	1.984686	0.184279	4.747221
Interaction	2	74.33333	37.16667	1.024502	0.388342	3.88529
Within	12	435.3333	36.27778			
Total	17	588				

At the 0.05 level of significance,

a) Is there an interaction between the cleansing step and the etching step? [5 marks]

b) Test if there is an effect due to the cleansing step? [5 marks]

c) Test if there is an effect due to the etching step? [5 marks]

d) Plot and estimate the mean yield for each cleansing step for the two etching steps? [5 marks]

e) Discuss the results of (a), (b), (c), and (d) above? [5 marks]

[Total: 25 marks]

End of Questions

STATISTICAL FORMULAE

A. DESCRIPTIVE STATISTICS

Sample Mean = $\overline{X} = \dfrac{\sum_{i=1}^{n} X_i}{n}$ Sample Standard Deviation = s = $\sqrt{\dfrac{\sum_{i=1}^{n} X_i^2}{n-1} - \dfrac{\left(\sum_{i=1}^{n} X_i\right)^2}{n(n-1)}}$

where n = *number of observations*
X_i = *the* i^{th} *observation of the data*

B. HYPOTHESIS TESTING

Types of Error
Type I Error = α= P(Rejecting H_0 | H_0 is true) where, Confidence Interval = 1 - α
Type II Error = β= P(Not Rejecting H_0 | H_0 is false)

One Sample Mean Test	
σ Known	**σ Unknown**
$z = \dfrac{\overline{x} - \mu}{\sigma/\sqrt{n}}$	$t = \dfrac{\overline{x} - \mu}{s/\sqrt{n}}$

Two Sample Mean Test	
Comparing Means for Two Independent Populations	
[Standard Deviation (σ) Known]	**[Standard Deviation (σ) Not Known]**
$z = \dfrac{(\overline{x}_1 - \overline{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}}$	$t = \dfrac{(\overline{x}_1 - \overline{x}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2\left(1/n_1 + 1/n_2\right)}}$ where $S_p^2 = \dfrac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{(n_1+n_2-2)}$

Two Sample Mean Test
Comparing Means for Two Paired Populations
$t = \dfrac{(\overline{D} - \mu_D)}{S_D/\sqrt{n}}$ where $\overline{D} = \dfrac{\sum_{i=1}^{n} D_i}{n}$ and $S_D = \sqrt{\dfrac{\sum_{i=1}^{n} D_i^2}{n-1} - \dfrac{\left(\sum_{i=1}^{n} D_i\right)^2}{n(n-1)}}$

Non-Parametric Analysis

Wilcoxon Rank Sum Test

$$Z = \frac{\left(T_1 - \mu_{T_1}\right)}{\sigma_{T_1}} \quad \text{where}$$

$$\mu_{T1} = \frac{n_1(n+1)}{2} \quad \text{and}$$

$$\sigma_{T_1} = \sqrt{\frac{n_1 n_2 (n+1)}{12}} \quad \text{where } n = n_1 + n_2$$

Wilcoxon Signed Rank Sum Test

$$Z = \frac{\left(T_+ - \mu_{T_+}\right)}{\sigma_{T_+}} \quad \text{where}$$

$$\mu_{T+} = \frac{n(n+1)}{4} \quad \text{and}$$

$$\sigma_{T_+} = \sqrt{\frac{n(n+1)(2n+1)}{24}}$$

Kruskal-Wallis Rank Test

$$H = \left[\frac{12}{n(n+1)} \sum_{j=1}^{c} \frac{T_j^2}{n_j} \right] - 3(n+1) \quad \text{where the critical value is } \chi^2 \text{ with } df = c - 1$$

Check ranking sum: $\sum T_j = n(n+1)/2$

Chi-Square Test

$$\chi^2 = \sum^{n} \frac{(O-E)^2}{E}$$

where O = Frequency of Observed Values and E = Frequency of Expected Values

with df = c – 1 where c = number of categories
or
with df = (r – 1)(c – 1) where r = number of rows and c = number of columns

C. ANALYSIS OF VARIANCE (ANOVA)

One-Way ANOVA

Source	Degrees of Freedom	Sum of Squares	Mean Squares	F-statistic
Among Groups	*c - 1*	SSA	MSA = SSA/*c-1*	MSA/MSW
Within Groups	*n - c*	SSW	MSW = SSW/*n-c*	
Total	*n - 1*	SST		

$$SST = \sum_{j=1}^{c}\sum_{i=1}^{n_j}\left(X_{ij} - \overline{\overline{X}}\right)^2 \quad \textit{or alternative formula:} \quad SST = \left(\sum_{j=1}^{c}\sum_{i=1}^{n_i} X_{ij}^2\right) - \frac{\left(\sum_{j=1}^{c}\sum_{i=1}^{n_i} X_{ij}\right)^2}{n}$$

$$SSA = \sum_{j=1}^{c} n_j\left(\overline{X}_j - \overline{\overline{X}}\right)^2 \quad \textit{and} \quad SSW = SST - SSA$$

where n = number of observations, c = number of groups and $\overline{\overline{X}}$ *= overall mean*

Tukey-Kramer Procedure

$$\text{Critical Range} = Q_U\sqrt{\frac{MSW}{2}\left[\frac{1}{n_i} + \frac{1}{n_j}\right]}$$

where Q_u *= the upper tail critical value from a Studentized Range Distribution having (c) degrees of freedom in the numerator and (n – c) degrees of freedom in the denominator at a given level of significance*

Two-Way ANOVA

Source	Degrees of Freedom	Sum of Squares	Mean Squares	F-statistic
A	$r-1$	SSA	MSA = SSA/$(r-1)$	MSA / MSE
B	$c-1$	SSB	MSB = SSB/$(c-1)$	MSB / MSE
AB	$(r-1)(c-1)$	SSAB	MSAB = SSAB/$(r-1)(c-1)$	MSAB / MSE
Error	$rc\,(n-1)$	SSE	MSE = SSE/$rc(n'-1)$	
Total	$n-1$	SST		

where,

Factor A levels are represented by the rows and Factor B levels are represented by the columns and
n = number of observations
c = number of columns
r = number of rows
n' = number of replicates

$$SST = \sum_{i=1}^{r}\sum_{j=1}^{c}\sum_{k=1}^{n'}\left(X_{ijk} - \overline{\overline{X}}\right)^2 \qquad SSA = cn'\sum_{i=1}^{r}\left(\overline{X}_i - \overline{\overline{X}}\right)^2$$

$$SSB = rn'\sum_{j=1}^{c}\left(\overline{X}j - \overline{\overline{X}}\right)^2 \qquad \text{where } \overline{\overline{X}} = \textit{overall mean}$$

$$SSE = (n'-1)\left[S_1^2 + S_2^2 + \ldots\ldots + S_k^2\right] \qquad \text{where } S_i^2 = \text{variance of each block}$$

D. REGRESSION ANALYSIS

Multiple Linear Regression

Population Model: $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots\ldots + \beta_k X_k + \varepsilon$

Sample Model: $\hat{y} = b_0 + b_1 x_1 + b_2 x_2 + \ldots\ldots + b_k x_k + e$

Adjusted R-Square $= 1 - \left[\dfrac{(1-R^2)(n-1)}{(n-p-1)}\right]$ where p = number of independent/predictor variables

ANOVA Table for Regression

Source	Degrees of Freedom	Sum of Squares	Mean Squares
Regression	p	SSR	MSR = SSR/p
Error/Residual	$n-p-1$	SSE	MSE = SSE/$(n-p-1)$
Total	$n-1$	SST	

Test Statistic for Significance of the Overall Regression Model

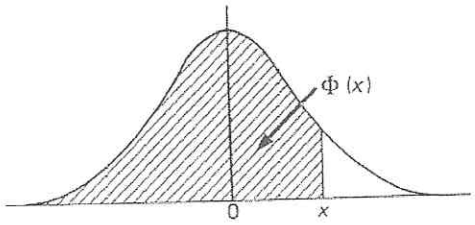
F = MSR/MSE

Test Statistic for Significance of Each Predictor Variable

$t_i = \dfrac{b_i}{S_{b_i}}$ and the critical value $= \pm t_{\alpha/2,(n-p-1)}$

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

The function tabulated is $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}t^2}\, dt$. $\Phi(x)$ is the probability that a random variable, normally distributed with zero mean and unit variance, will be less than or equal to x. When $x < 0$ use $\Phi(x) = 1 - \Phi(-x)$, as the normal distribution with zero mean and unit variance is symmetric about zero.



x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
0·00	0·5000	0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·97725
·01	·5040	·41	·6591	·81	·7910	·21	·8869	·61	·9463	·01	·97778
·02	·5080	·42	·6628	·82	·7939	·22	·8888	·62	·9474	·02	·97831
·03	·5120	·43	·6664	·83	·7967	·23	·8907	·63	·9484	·03	·97882
·04	·5160	·44	·6700	·84	·7995	·24	·8925	·64	·9495	·04	·97932
0·05	0·5199	0·45	0·6736	0·85	0·8023	1·25	0·8944	1·65	0·9505	2·05	0·97982
·06	·5239	·46	·6772	·86	·8051	·26	·8962	·66	·9515	·06	·98030
·07	·5279	·47	·6808	·87	·8078	·27	·8980	·67	·9525	·07	·98077
·08	·5319	·48	·6844	·88	·8106	·28	·8997	·68	·9535	·08	·98124
·09	·5359	·49	·6879	·89	·8133	·29	·9015	·69	·9545	·09	·98169
0·10	0·5398	0·50	0·6915	0·90	0·8159	1·30	0·9032	1·70	0·9554	2·10	0·98214
·11	·5438	·51	·6950	·91	·8186	·31	·9049	·71	·9564	·11	·98257
·12	·5478	·52	·6985	·92	·8212	·32	·9066	·72	·9573	·12	·98300
·13	·5517	·53	·7019	·93	·8238	·33	·9082	·73	·9582	·13	·98341
·14	·5557	·54	·7054	·94	·8264	·34	·9099	·74	·9591	·14	·98382
0·15	0·5596	0·55	0·7088	0·95	0·8289	1·35	0·9115	1·75	0·9599	2·15	0·98422
·16	·5636	·56	·7123	·96	·8315	·36	·9131	·76	·9608	·16	·98461
·17	·5675	·57	·7157	·97	·8340	·37	·9147	·77	·9616	·17	·98500
·18	·5714	·58	·7190	·98	·8365	·38	·9162	·78	·9625	·18	·98537
·19	·5753	·59	·7224	·99	·8389	·39	·9177	·79	·9633	·19	·98574
0·20	0·5793	0·60	0·7257	1·00	0·8413	1·40	0·9192	1·80	0·9641	2·20	0·98610
·21	·5832	·61	·7291	·01	·8438	·41	·9207	·81	·9649	·21	·98645
·22	·5871	·62	·7324	·02	·8461	·42	·9222	·82	·9656	·22	·98679
·23	·5910	·63	·7357	·03	·8485	·43	·9236	·83	·9664	·23	·98713
·24	·5948	·64	·7389	·04	·8508	·44	·9251	·84	·9671	·24	·98745
0·25	0·5987	0·65	0·7422	1·05	0·8531	1·45	0·9265	1·85	0·9678	2·25	0·98778
·26	·6026	·66	·7454	·06	·8554	·46	·9279	·86	·9686	·26	·98809
·27	·6064	·67	·7486	·07	·8577	·47	·9292	·87	·9693	·27	·98840
·28	·6103	·68	·7517	·08	·8599	·48	·9306	·88	·9699	·28	·98870
·29	·6141	·69	·7549	·09	·8621	·49	·9319	·89	·9706	·29	·98899
0·30	0·6179	0·70	0·7580	1·10	0·8643	1·50	0·9332	1·90	0·9713	2·30	0·98928
·31	·6217	·71	·7611	·11	·8665	·51	·9345	·91	·9719	·31	·98956
·32	·6255	·72	·7642	·12	·8686	·52	·9357	·92	·9726	·32	·98983
·33	·6293	·73	·7673	·13	·8708	·53	·9370	·93	·9732	·33	·99010
·34	·6331	·74	·7704	·14	·8729	·54	·9382	·94	·9738	·34	·99036
0·35	0·6368	0·75	0·7734	1·15	0·8749	1·55	0·9394	1·95	0·9744	2·35	0·99061
·36	·6406	·76	·7764	·16	·8770	·56	·9406	·96	·9750	·36	·99086
·37	·6443	·77	·7794	·17	·8790	·57	·9418	·97	·9756	·37	·99111
·38	·6480	·78	·7823	·18	·8810	·58	·9429	·98	·9761	·38	·99134
·39	·6517	·79	·7852	·19	·8830	·59	·9441	·99	·9767	·39	·99158
0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·9772	2·40	0·99180

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
2·40	0·99180	2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918
·41	·99202	·56	·99477	·71	·99664	·86	·99788	·01	·99869	·16	·99921
·42	·99224	·57	·99492	·72	·99674	·87	·99795	·02	·99874	·17	·99924
·43	·99245	·58	·99506	·73	·99683	·88	·99801	·03	·99878	·18	·99926
·44	·99266	·59	·99520	·74	·99693	·89	·99807	·04	·99882	·19	·99929
2·45	0·99286	2·60	0·99534	2·75	0·99702	2·90	0·99813	3·05	0·99886	3·20	0·99931
·46	·99305	·61	·99547	·76	·99711	·91	·99819	·06	·99889	·21	·99934
·47	·99324	·62	·99560	·77	·99720	·92	·99825	·07	·99893	·22	·99936
·48	·99343	·63	·99573	·78	·99728	·93	·99831	·08	·99896	·23	·99938
·49	·99361	·64	·99585	·79	·99736	·94	·99836	·09	·99900	·24	·99940
2·50	0·99379	2·65	0·99598	2·80	0·99744	2·95	0·99841	3·10	0·99903	3·25	0·99942
·51	·99396	·66	·99609	·81	·99752	·96	·99846	·11	·99906	·26	·99944
·52	·99413	·67	·99621	·82	·99760	·97	·99851	·12	·99910	·27	·99946
·53	·99430	·68	·99632	·83	·99767	·98	·99856	·13	·99913	·28	·99948
·54	·99446	·69	·99643	·84	·99774	·99	·99861	·14	·99916	·29	·99950
2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918	3·30	0·99952

The critical table below gives on the left the range of values of x for which $\Phi(x)$ takes the value on the right, correct to the last figure given; in critical cases, take the upper of the two values of $\Phi(x)$ indicated.

3·075	0·9990	3·263	0·9994	3·731	0·99990	3·916	0·99995
3·105	0·9991	3·320	0·9995	3·759	0·99991	3·976	0·99996
3·138	0·9992	3·389	0·9996	3·791	0·99992	4·055	0·99997
3·174	0·9993	3·480	0·9997	3·826	0·99993	4·173	0·99998
3·215	0·9994	3·615	0·9998	3·867	0·99994	4·417	0·99999
			0·9999		0·99995		1·00000

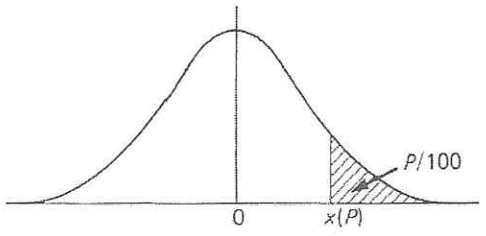
When $x > 3{\cdot}3$ the formula $1-\Phi(x) \doteqdot \frac{e^{-\frac{1}{2}x^2}}{x\sqrt{2\pi}}\left[1-\frac{1}{x^2}+\frac{3}{x^4}-\frac{15}{x^6}+\frac{105}{x^8}\right]$ is very accurate, with relative error less than $945/x^{10}$.

TABLE 5. PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

This table gives percentage points $x(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{2\pi}}\int_{x(P)}^{\infty} e^{-\frac{1}{2}t^2}\,dt.$$

If X is a variable, normally distributed with zero mean and unit variance, $P/100$ is the probability that $X \geqslant x(P)$. The lower P per cent points are given by symmetry as $-x(P)$, and the probability that $|X| \geqslant x(P)$ is $2P/100$.



P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$
50	0·0000	5·0	1·6449	3·0	1·8808	2·0	2·0537	1·0	2·3263	0·10	3·0902
45	0·1257	4·8	1·6646	2·9	1·8957	1·9	2·0749	0·9	2·3656	0·09	3·1214
40	0·2533	4·6	1·6849	2·8	1·9110	1·8	2·0969	0·8	2·4089	0·08	3·1559
35	0·3853	4·4	1·7060	2·7	1·9268	1·7	2·1201	0·7	2·4573	0·07	3·1947
30	0·5244	4·2	1·7279	2·6	1·9431	1·6	2·1444	0·6	2·5121	0·06	3·2389
25	0·6745	4·0	1·7507	2·5	1·9600	1·5	2·1701	0·5	2·5758	0·05	3·2905
20	0·8416	3·8	1·7744	2·4	1·9774	1·4	2·1973	0·4	2·6521	0·01	3·7190
15	1·0364	3·6	1·7991	2·3	1·9954	1·3	2·2262	0·3	2·7478	0·005	3·8906
10	1·2816	3·4	1·8250	2·2	2·0141	1·2	2·2571	0·2	2·8782	0·001	4·2649
5	1·6449	3·2	1·8522	2·1	2·0335	1·1	2·2904	0·1	3·0902	0·0005	4·4172

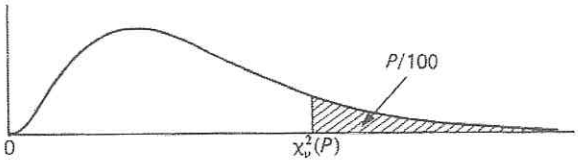
TABLE 8. PERCENTAGE POINTS OF THE χ^2-DISTRIBUTION

This table gives percentage points $\chi^2_\nu(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{2^{\nu/2}\,\Gamma(\frac{\nu}{2})} \int_{\chi^2_\nu(P)}^{\infty} x^{\frac{1}{2}\nu-1}\, e^{-\frac{1}{2}x}\, dx.$$

If X is a variable distributed as χ^2 with ν degrees of freedom, $P/100$ is the probability that $X \geqslant \chi^2_\nu(P)$.

For $\nu > 100$, $\sqrt{2X}$ is approximately normally distributed with mean $\sqrt{2\nu-1}$ and unit variance.



(The above shape applies for $\nu \geqslant 3$ only. When $\nu < 3$ the mode is at the origin.)

P	50	40	30	20	10	5	2·5	1	0·5	0·1	0·05
$\nu = 1$	0·4549	0·7083	1·074	1·642	2·706	3·841	5·024	6·635	7·879	10·83	12·12
2	1·386	1·833	2·408	3·219	4·605	5·991	7·378	9·210	10·60	13·82	15·20
3	2·366	2·946	3·665	4·642	6·251	7·815	9·348	11·34	12·84	16·27	17·73
4	3·357	4·045	4·878	5·989	7·779	9·488	11·14	13·28	14·86	18·47	20·00
5	4·351	5·132	6·064	7·289	9·236	11·07	12·83	15·09	16·75	20·52	22·11
6	5·348	6·211	7·231	8·558	10·64	12·59	14·45	16·81	18·55	22·46	24·10
7	6·346	7·283	8·383	9·803	12·02	14·07	16·01	18·48	20·28	24·32	26·02
8	7·344	8·351	9·524	11·03	13·36	15·51	17·53	20·09	21·95	26·12	27·87
9	8·343	9·414	10·66	12·24	14·68	16·92	19·02	21·67	23·59	27·88	29·67
10	9·342	10·47	11·78	13·44	15·99	18·31	20·48	23·21	25·19	29·59	31·42
11	10·34	11·53	12·90	14·63	17·28	19·68	21·92	24·72	26·76	31·26	33·14
12	11·34	12·58	14·01	15·81	18·55	21·03	23·34	26·22	28·30	32·91	34·82
13	12·34	13·64	15·12	16·98	19·81	22·36	24·74	27·69	29·82	34·53	36·48
14	13·34	14·69	16·22	18·15	21·06	23·68	26·12	29·14	31·32	36·12	38·11
15	14·34	15·73	17·32	19·31	22·31	25·00	27·49	30·58	32·80	37·70	39·72
16	15·34	16·78	18·42	20·47	23·54	26·30	28·85	32·00	34·27	39·25	41·31
17	16·34	17·82	19·51	21·61	24·77	27·59	30·19	33·41	35·72	40·79	42·88
18	17·34	18·87	20·60	22·76	25·99	28·87	31·53	34·81	37·16	42·31	44·43
19	18·34	19·91	21·69	23·90	27·20	30·14	32·85	36·19	38·58	43·82	45·97
20	19·34	20·95	22·77	25·04	28·41	31·41	34·17	37·57	40·00	45·31	47·50
21	20·34	21·99	23·86	26·17	29·62	32·67	35·48	38·93	41·40	46·80	49·01
22	21·34	23·03	24·94	27·30	30·81	33·92	36·78	40·29	42·80	48·27	50·51
23	22·34	24·07	26·02	28·43	32·01	35·17	38·08	41·64	44·18	49·73	52·00
24	23·34	25·11	27·10	29·55	33·20	36·42	39·36	42·98	45·56	51·18	53·48
25	24·34	26·14	28·17	30·68	34·38	37·65	40·65	44·31	46·93	52·62	54·95
26	25·34	27·18	29·25	31·79	35·56	38·89	41·92	45·64	48·29	54·05	56·41
27	26·34	28·21	30·32	32·91	36·74	40·11	43·19	46·96	49·64	55·48	57·86
28	27·34	29·25	31·39	34·03	37·92	41·34	44·46	48·28	50·99	56·89	59·30
29	28·34	30·28	32·46	35·14	39·09	42·56	45·72	49·59	52·34	58·30	60·73
30	29·34	31·32	33·53	36·25	40·26	43·77	46·98	50·89	53·67	59·70	62·16
32	31·34	33·38	35·66	38·47	42·58	46·19	49·48	53·49	56·33	62·49	65·00
34	33·34	35·44	37·80	40·68	44·90	48·60	51·97	56·06	58·96	65·25	67·80
36	35·34	37·50	39·92	42·88	47·21	51·00	54·44	58·62	61·58	67·99	70·59
38	37·34	39·56	42·05	45·08	49·51	53·38	56·90	61·16	64·18	70·70	73·35
40	39·34	41·62	44·16	47·27	51·81	55·76	59·34	63·69	66·77	73·40	76·09
50	49·33	51·89	54·72	58·16	63·17	67·50	71·42	76·15	79·49	86·66	89·56
60	59·33	62·13	65·23	68·97	74·40	79·08	83·30	88·38	91·95	99·61	102·7
70	69·33	72·36	75·69	79·71	85·53	90·53	95·02	100·4	104·2	112·3	115·6
80	79·33	82·57	86·12	90·41	96·58	101·9	106·6	112·3	116·3	124·8	128·3
90	89·33	92·76	96·52	101·1	107·6	113·1	118·1	124·1	128·3	137·2	140·8
100	99·33	102·9	106·9	111·7	118·5	124·3	129·6	135·8	140·2	149·4	153·2